Times Tables - Business as usual or a better balance?

Report of a study into the effectiveness of a more "conceptual approach to teaching multiplication facts

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Background and context

Having a deep understanding of number bonds to 10 and multiplication facts and the ability to use them as part of a strategy to calculate using larger numbers are important aspects of becoming a successful mathematician for children in primary schools and beyond. An OFSTED study of twenty schools which were successful in teaching mathematics (OFSTED, 2011) suggested that weakness in these and in understanding place value would impede all methods of calculation. Those schools, "ensured that all pupils have a good grasp of number facts, structures and concepts".

There are, however, different conceptions of what it means to be "fluent" with times tables.

We might, for the sake of discussion, term two of the contrasting approaches "conceptual" and "procedural", where a conceptual approach emphasises understanding of the connections and patterns which make up the multiplication facts – repeated addition and arrays of different forms and their application to a range of problem types and a procedural approach emphasises memorisation and quick recall of the facts.

The National Curriculum takes a balanced approach.

It suggests that children become fluent "…so that pupils have conceptual understanding and are able to recall and apply their knowledge rapidly and accurately to problems" (DfE, 2013) and says of times tables that children should learn how they are connected to each other and to other mathematical ideas such as place value.

However, it also states that pupils should have '**memorised** their multiplication tables up to and including the 12 multiplication table' by the age of 9" (DfE, 2013), suggesting that its authors see value in both approaches.

Politicians, including previous and recent Secretaries of State for Education, have hit upon this aspect of understanding multiplication facts with one arguing that, as part of the "war on innumeracy … We will expect every pupil by the age of 11 to know their times tables off by heart" (Sunday Times, 2015).

A compulsory national times tables test for children in Year 6 may or may not be introduced in the next two years.

Traditionally, schools have concentrated on speed and memory in teaching times tables. From weekly (and in the run up to the SAT tests, sometimes daily!) tables tests, to "speedy tables", to website games, to activities and games such as "product partners" and "POW", to posters on the wall and even chanting tables in unison.

But some teachers consider that these, "speed and memory" approaches only work well for some children. It appears that quite a lot of children just don't seem to be able to commit tables to their long term memory.

Jo Boaler, Professor of Mathematics Education at the Stanford Graduate School of Education (2015), suggests that speed and memory activities are not the best way for children to become fluent in their understanding of multiplication facts.

She is of the opinion that it is more important to develop "number sense" rather than memory (Boaler, 2015) and even suggests that timed tests can discourage children from learning in maths for life and lead to "maths anxiety" (Beilock, 2011).

She suggests developing a more conceptual approach by replacing timed tests with games and activities based around seeing the patterns and connections involved in times tables such as a "Snap" type game (where children are asked to link the same number fact e.g. 3 x 8 with the same fact shown in different ways e.g. as an array, a 3 x 8 table, 24 ÷ 3, 24 ÷ 8, 24, 3 jumps of 8 on a number line, 8 jumps of 3 etc), a game based on arrays called "How close to 100" and a dice game called "Pepperoni Pizza" where children are asked to multiply the number of pizzas by the number of pieces of pepperoni on each and to say the tables fact that it represents.

We considered that this would be a good area, at an opportune time, to study.

Methodology

Twenty - eight schools in the North West of England were recruited for the study initially. They were recruited by e mail invitation from the North West Maths Hubs and from the Math Subject Leaders' meeting in Sefton borough.

All of the schools were Primary schools. There was a large variation in the percentage of children in receipt of free school meals (Fig.1)

Code	FSM %	Code	FSM %
1	3	12	20
2	50	13	23
3	6	14	4
4	20	15	23
5	0	16	12
6	10	17	4
7	10	18	30
8	11	19	42
9	12	20	7
10	17	21	13
11	70	22	10

Fig.1

It was decided to study children in Year 3 as there are definite expectations in the National Curriculum of which tables the children in Year 3 are expected to know – the 3, 4 and 8 times tables.

Children were given two tests (Appendix 1). One was designed to be a traditional speed and memory test in which children were asked to answer a question such as 4 x 6, the question was repeated once and the children then had seven seconds to write the answer. The other was designed as a more, "conceptual" test in which children were asked to link arrays of different types and images of repeated addition to the connected tables fact.

The tests were given in the week beginning 20th March 2017 and the scores recorded by the class teacher. The same tests were then given to the classes at the end of the study in the week beginning 10th July 2017.

Thus, the three factors gave rise to "matched groups" in which there were broadly even numbers of schools with high and low pre test scores and high and low percentages of children in receipt of free school meals in each group which became a "Treatment" group and a "Control" group (Fig2,3 and 4)

Group comparison – Free School Meals

Treatment Schools			Control Schools	
Code	FSM %		Code	FSM %
1	3		12	20
2	50		13	23
3	6		14	4
4	20		15	23
5	0		16	12
6	10		17	4
7	10		18	30
8	11		19	42
9	12		20	7
10	17		21	13
11	70		22	10

Fig.2

Initial Test scores

Test 1				
Treatment Schools			Control Schools	
Code	Average score		Code	Average Score
1	3.4		12	4.3
2	2.3		13	1.8
3	3.0		14	4.8
4	3.5		15	5.8
5	3.2		16	3.9
6	3.7		17	4.0
7	3.5		18	3.2
8	3.4		19	3.4
9	3.8		20	4.3
10	3.7		21	4.4
11	2.6		22	4.1

Fig.3

Test 2				
Treatment Schools			Control Schools	
Code	Average score		Code	Average Score
1	5.8		12	6.8
2	3.8		13	3.9
3	4.3		14	5.8
4	6.4		15	8
5	4.2		16	5.3
6	6.5		17	5.3
7	6.3		18	3.4
8	3.8		19	5.1
9	4.1		20	5.3
10	5.4		21	5.5
11	5.1		22	5.1

Fig.4

The inclusion of the percentage of children in receipt of free school meals was used as if, without including this factor, schools in either the control or treatment groups had a great imbalance of children with FSM, it could affect the result.

The matching process could be a limiting factor in the study. Rather than just one measure, it was considered that the effect on both speed tests and more conceptual tests was important – would a more conceptual approach only affect the children's ability to answer questions set in a conceptual way or would it also mean that "recall" of tables facts was improved?

Teachers in the Treatment group were given a booklet with the instructions for a variety of Snap games using cards (six cards for each tables fact showing the fact, the product and a range of other representations – arrays of different symbols and pictures, representations of repeated addition etc), "How close to 100?", "Pepperoni Pizza" and "Bucket and Ring" – games and activities which required children to explore the concepts behind the tables facts and to link them to a range of images.

They were asked to replace what they normally did to teach times tables with these materials using them at least three times a week.

The Control group were asked to carry on in the same way that they usually did in teaching tables – business as usual.

Fidelity

All of the Treatment schools were visited once in the term and classes were observed using the materials. There was little ambiguity about how the materials were to be used but these visits were useful in showing that some schools had made an earlier start with the materials and were more proactive in their use (Appendix 2)

Ethics

No children or schools could be identified individually in the study except by the study leader. A standard letter was sent to schools to send to parents and carers of children in the classes used explaining what the study was and explaining that they could withdraw their child from the study at any time. Schools and individual class teachers could withdraw from the study at any time. All data and records were kept secure.

Results

The control group had slightly higher pre - test scores on average than the treatment schools and the treatment schools had slightly higher percentages of children in receipt of free school meals (fig2,3,4)

The data from some of the schools was not included in the final study as they did not provide final test data by the end of the Summer Term.

Thus, the data from 22 (11 in each group) schools was included in the final study.

Anecdotal evidence from the fidelity visits (Appendix 2) indicated that the children overwhelmingly found that the children in the treatment group enjoyed the activities and teachers reported that the children were more engaged in tables learning when using them.

Using the Education Endowment Foundation's "DIY Evaluation Guide" (EEF, 2013) "effect sizes" for both tests were calculated (Fig.5)

Effect sizes are quantitative measures of the size and consistency of the impact on an outcome, in this case attainment in times tables.

The process for each test was:

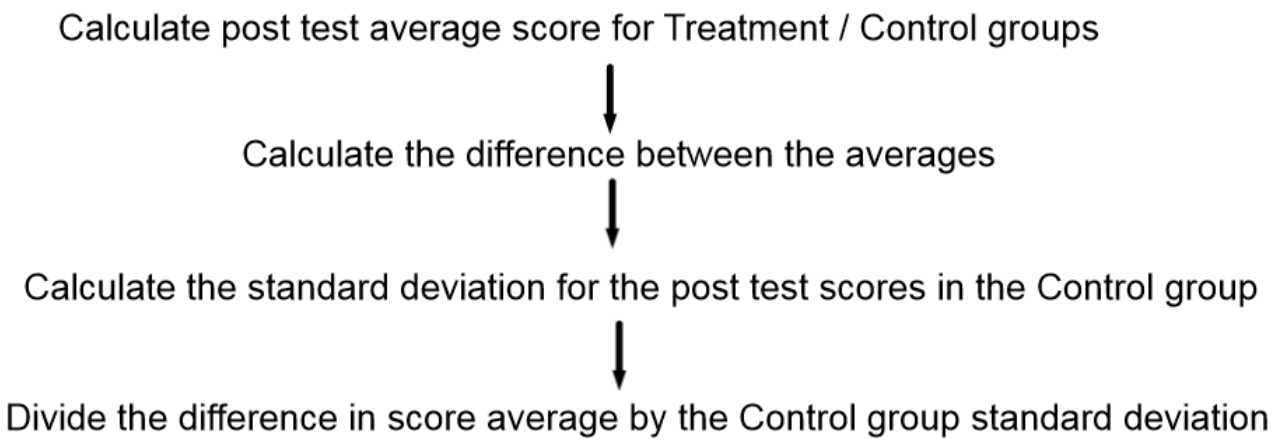

Fig.5

Effect size calculation Test 1

School Treatment group	Post Test 1 scores	School Control group	Post Test 1 scores
1	4.9	12	5.0
2	2.9	13	2.3
3	5.4	14	5.1
4	3.7	15	6.4
5	5.2	16	.2
6	4.8	17	4.1
7	4.4	18	3.7
8	5.5	18	3.7
9	6.0	20	5.1
10	4.5	21	4.6
11	4.2	22	4.2
AVERAGE	4.681		4.309

Difference between the averages	0.372
Standard deviation of control group	1.106
Difference in average score ÷ Standard Deviation of control group	0.375 ÷ 1.106
Effect Size	**0.339**

An effect size of 0.339 is considered by the EEF to be “moderate” and to equate to a positive effect of the intervention of 4 months

Effect size calculation Test 2

School Treatment group	Post Test 2 scores	School Control group	Post Test 2 scores
1	7.5	12	7.2
2	4.8	13	4.8
3	6.5	14	6.8
4	6.5	15	8.5
5	6.0	16	5.9
6	7.5	17	6.3
7	6.3	18	4.3
8	7.1	18	6.7
9	6.8	20	6.6
10	6.4	21	6.1
11	6.0	22	5.4
AVERAGE	6.490		6.236

Difference between the averages	0.254
Standard deviation of control group	1.156
Difference in average score ÷ Standard Deviation of control group	0.254 ÷ 1.156
Effect Size	**0.220**

An effect size of 0.220 is considered by the EEF to be “moderate” and to equate to a positive effect of the intervention of 3 months

As well as the effect size, the average increase in test scores between the pre test scores and the post test scores for each test was calculated. This was a secondary evaluation measure because taking into account three measures in the matching process (Test scores 1 and 2 and free school meals) meant that the Control group and the Treatment group had varying score averages on the individual tests.

The average improvement in scores for the Treatment group on Test 1 was 1.3 marks and 0.3 for the control group. For Test 2, the average score improvement for the Treatment group was 1.4 and for the control group, it was 0.8 (Fig.6)

Fig.6

Change in scores in Test 1 pre and post test			
Treatment		Control	
Code	Change T1	Code	Change T1
1	1.5	12	0.7
2	0.6	13	0.5
3	2.4	14	0.3
4	0.2	15	0.6
5	2.0	16	-0.7
6	0.8	17	0.1
7	0.9	18	0.5
8	2.1	19	0.3
9	2.2	20	0.8
10	0.8	21	0.2
11	1.6	22	0.1
Average	1.3		0.3

Change in scores in Test 2 pre and post test			
Treatment		Control	
Code	Change T2	Code	Change T2
1	1.7	12	0.4
2	1	13	0.9
3	2.2	14	1.0
4	0.1	15	0.5
5	1.8	16	0.6
6	1	17	1.0
7	1	18	0.9
8	3.3	19	1.6
9	2.7	20	1.3
10	1	21	0.6
11	0.9	22	0.3
Average	1.5		0.8

Discussion

The study results suggest that using the more conceptual approach and materials to teach times tables has a moderate positive effect on success in traditional tables tests as well as more conceptual tests.

The effect size recorded was commensurate but slightly lower for the conceptual tests, possibly reflecting that some schools are incorporating more problem-solving and reasoning into their daily mathematics teaching.

The difference in the rise in average scores for both tests, given that the control group began with slightly higher average scores, further supports a moderate positive effect for the approach.

The National Curriculum lays out a balanced approach to the teaching of multiplication facts but many, if not most, schools continue to use speed and memory approaches in relation to specifically teaching times tables. It is not uncommon, on asking a teacher how they teach times tables, to be answered that they have weekly, or even daily, tests – as if testing and teaching were the same activity.

With times tables, modelling and direct explicit instruction does not have to be limited to repetitive procedural activities which aim to get children to memorise the tables. This may work for some children but "schemas" – internal models of understanding - for other children may be constructed more effectively by modelling the connections and patterns between the elements of the tables facts – repeated addition and arrays of different types alongside more procedural, "practising" activities.

Asking children to "play games" with the materials might suggest that children are asked to "discover" the connections for themselves but the instruction booklet asked teachers in the treatment schools to specifically explain the connections with worked examples. In the fidelity visits (Appendix 2), it was clear that most teachers had done this very effectively before asking the children to use the materials in groups.

Some argue that the prevalence of timed tests for times tables is one of the factors which have led to the development of a high incidence of maths anxiety in the UK (Beilock, 2011, Boaler, 2015).

Wherever you stand on this issue, there is clearly a strong argument for using a wider range of teaching approaches than frequent timed tests.

Conclusion

Using materials to teach times tables which help children to understand the connections and patterns inherent in the tables seems to have a moderate positive effect on both their conceptual understanding and their ability to correctly answer questions in a timed test.

A better balance of approaches designed to help children to set the concepts of tables into their long term memory might be more effective than the procedural, speed and memory approaches widely used currently alone.

Using a balance of good quality procedural activities which allow children to practice tables facts in such a way as is enjoyable and more conceptual activities, such as the materials used in this study, with less frequent timed testing (perhaps half termly for formative assessment) might lead to greater success for a greater number of pupils.

It is suggested that a wider study, perhaps trialling the effect of an approach which balances these two aspects into a useable classroom product is desirable.

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Cronton C of E Primary School.
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Bishop Martin C of E Primary School, Liverpool.
Litherland Moss Primary School, Sefton.
English Martyrs Catholic Primary School, Sefton.
The Beacon C of E Primary School, Liverpool.
Valewood Primary School, Sefton.
St Monica's Catholic Primary School, Sefton.
St Robert Bellarmine Catholic Primary School, Bootle.
Ursuline Catholic Primary School, Sefton.
Ash Grove Academy, Macclesfield.
St Mark's Catholic Primary School, Liverpool.
Larkfield Primary School, Southport.
Green Park Primary School, Liverpool.
St Nicholas C of E Primary School, Liverpool.
St Edmund's and St Thomas' Catholic Primary School, Sefton.
St Albans Catholic Primary School, Macclesfield.
Netherton Moss Primary School, Bootle.
Hudson Primary School, Liverpool

Appendix 1- Procedural (Test 1) and Conceptual (Test 2) Tests

(Reduced in size for convenience)

Test 1

Say "I am going to ask a times tables question and I want you to just write down the answer next to the letter of the question. So I might say a) is 2 x 2 and you would write down the answer next to the letter. Do not write down the question. You will have seven seconds to answer each question. If you don't manage to write the answer down, don't worry, just move on to the next one".

Read each question once and then repeat it and then give 7 seconds before moving on to the next question.

a) 3 x 4 =

b) 4 x 6 =

c) 8 x 3 =

d) 3 x 7 =

e) 4 x 8 =

f) 7 x 8 =

g) 9 x 3 =

h) 4 x 7 =

i) 6 x 8 =

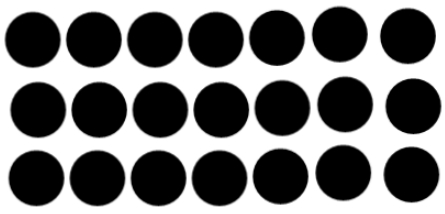
Test 2

a) $3 \times \square = 12$

b) 4 friends each pay £7 to go to the cinema. How much do they pay altogether?

£ _____

c) Use this array to complete these number sentences



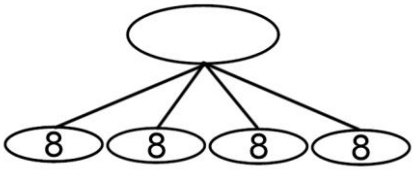
$\square \times 3 = \square$

$3 \times \square = \square$

$\square \div 3 = \square$

$\square \div \square = 3$

d) Fill in the missing number



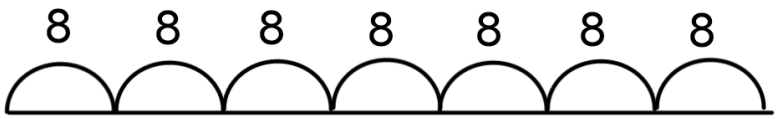
e) What could ? be?

$$? + ? + ? + ? = 24$$

? =_____

f) $6 \times 8 = 48 \div \square$

g) Write a multiplication sentence for this picture



$\square \times \square = \square$

Appendix 2 – Fidelity visits

Fidelity Visits

School 1

Two groups went out to work on the games with TAs using different tables. They were playing a snap game. The teacher said that they usually used just one table but were mixing them today. They had used a range of the activities over the period. She said that they had really enjoyed the games. As they played, she got them to explain the connections. She then introduced the dice game where they roll two dice and find the cards that fit that tables fact. As I talked to them, they could explain the connections well.

School 2

The teacher said that they had played them a few times but not as much as they wanted to. They would be playing them more from now on as they seemed to be popular.

The children said that they were enjoying the games.

The children were a bit confused as to how to play the snap games. Some were doing Pepperoni Pizza and seemed to be enjoying it.

School 3

Children played in 4s at desks. They played a Snap game and then moved on to play Pepperoni Pizza. They seemed used to playing the games regularly as they were very independent with them. I asked several what they thought of the games. One said that it was better than doing a tables test because "when you enjoy it you can understand it easier"

School 4

The teacher said that they had not done them much for a particular reason. She was going to be doing them more from now as the problem that was stopping them had now been overcome.

They were doing Find Someone Who but the matches were on tables for them to find. Once they had found a match, they found someone else who had one, making a pair with four cards. They then had to find the third person to make up the six images showing the same fact.

School 5

Children were very enthusiastic when the class teacher said that they were going to be playing the games. They played one of the snap games and, when I asked, said that they had also played Find Someone Who…

In future versions of the cards, it would be helpful to put a line to denote 6 or 9.

The teacher gave some input with examples at the beginning i.e. asking "What would match with this one…" and asked them to explain why. She emphasised that they should explain to each other in the game why they match.

When I was watching the games, children could tell me why the cards matched. They showed good reasoning.

School 6

They had had some whole school events which had prevented them from doing the activities as much as they would like but had used them over the weeks and the teacher said that the children really liked playing them. They were playing the "pelmanism" type game – finding cards that matched theirs and then putting it down and trying to remember where other players had put the ones that matched with theirs.

They had played some of the other games such as the snap ones.

The children said that the pelmanism one was their favourite.

School 7

They had been playing the games regularly, concentrating on the snap games which they were playing today. They were enjoying the games and said that the snap games were their favourites.

School 8

The teacher said that they had only played snap games because the other games such as Find someone who… would be too loud for this class. They played in 4s. They hadn't done the activities much. Each group had a lot of cards and so there were few snaps. I showed the teacher and some children the snap where you have fewer cards and you could see them and then decide whether to snap. We played a game and that seemed more satisfying to them

School 9

Different groups were playing different games – Snap (holding version), Pepperoni Pizza, How close and two groups were playing Bucket and ring in the class.

Some had tables grids for support.

In Pepperoni Pizza =, they were writing all of the facts for the pizza i.e. 3x4, 4x3, 12÷4, 12÷3.

The class teacher told me that she had been doing some pupil voice and several had said that the best thing about being in Year 3 had been the tables games.

The role of the TA was important. She understood the purpose of the games and knew to get them to explain the connections and she decided when some groups changed the size of the dice.

School 10

Children were very enthusiastic when the class teacher said that they were going to be playing the games. The teacher asked the children to explain the snap game that they would be playing. They did so really well.

They played the game in fours.

They were able to tell when, when I asked, what the connections between the images were. They were discussing the connections a lot in playing the game.

It seemed that the role of the TA is very important. If the know what the purpose of the games and activities are, they can "finesse" them e.g ask children to explain more or move on to a different set of cards.

I asked children if they preferred this approach or what they usually do for tables. They preferred this to tables tests.

They liked the snap games better than the others but said that they liked them all.

They could really explain to me the connections using words such as inverse and connection.

School 11

The teacher was working with a group. They played one of the snap games. They were quite independent with the game but the teacher was also stopping hem occasionally and setting up misconceptions such as Do you think these two form a pair? I don't think that they do because they look really different". The children then needed to explain the connection. He also held one card up and asked how they might represent it.

I thought that could be a different activity. Once they are familiar with the different representations on the cards, a larger group or whole class could make their own representations showing repeated addition or different types of array.